The early modern period saw the rapid development of two fundamental bodies of knowledge, astronomy and mechanics. The integration of these two bodies of knowledge later resulted in one of the most successful scientific theories, Newtonian classical mechanics. One of the first major steps in this development was Galileo’s attempt to present a coherent world picture linking Copernican astronomy with his new theory of motion, presented in the *Dialogue* of 1632. Although this attempt fell, in hindsight, short of its ambitions, it does testify to the fertility of integrating these two bodies of knowledge and has hence given rise to questions about which of them was the driving force. Did Galileo develop a new mechanics as a strategic plot in order to justify Copernican astronomy? Or did, on the contrary, his mechanical thinking necessitate his adherence to Copernicanism? Consequently Galileo’s ambitious attempt, first published in the *Dialogue*, to join a key insight of his mechanics, the law of fall, with the most advanced astronomical data in an attempt to explain the cosmogony of the planetary system appears to be uniquely suited to discuss these questions.

In a short passage in the *Dialogue* Galileo sketches this cosmogonical hypothesis and claims to have based calculations on it that agree “truly wonderfully” with observations.¹ Six years later in the *Discorsi* the hypothesis is reiterated.² The existing manuscript evidence of Galileo’s work on

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cosmogony has, however, received considerably less attention than the pas-
sages in the published work.

This paper presents some of the results of a new interpretation of six folio pages from a manuscript comprising Galileo’s notes on motion which is preserved as Ms. Gal 72 in the Biblioteca Nazionale Centrale in Florence. The diagrams and calculations scattered over the pages pertain to Galileo’s cosmogonical hypothesis. This paper will focus on showing that Galileo attempted to prove the empirical adequacy of this hypothesis when his new science of motion was still in its infancy and not yet sufficiently developed to tackle such a complex problem. Such a positioning of the work on the cosmogonical hypothesis to a time when Galileo probably was neither firmly committed to Copernicanism nor had a fully developed science of motion promises to shed new light on the interrelations between his mechanical and astronomical thinking.

In the Dialogue Galileo introduces an idea concerning the genesis of the planetary system, crediting Plato with its authorship. According to this idea, the “divine Architect” created the sun and, at a certain distance from it, the planets. The planets, according to their “assigned tendencies”, then began to fall towards the sun in naturally accelerated motion. Upon reaching their predestined orbits, their linear motions were diverted into circular motions by the “divine Mind”, thereby retaining their acquired velocities. Galileo’s spokesman Salviati raises the question of whether all planets could have been created in the same place –referred to in the following as the creation point– in order to account for the observed orbital velocities of the planets. After a fairly detailed description of how this hypothesis would have to be tested, Galileo informs us that he has carried out the required computations and that they agree “truly wonderfully” with his observations. When the topic is brought up again six years later in the Discorsi, Salviati once more emphasizes that Galileo had done the computation and “found it to answer very closely to the observations”.

These two passages in Galileo’s major works provoked great interest among his contemporaries and have also been discussed by historians of science. Intrigued by the obvious falsity of Galileo’s claim, first noticed by Mersenne in 1637 (Mersenne, M., 1637, pp. 103-107), historians have tried to come to terms with the motives for Galileo’s insistence on his cosmogonical hypothesis. Interpretive attempts focused initially on reconstructing

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4 This reiteration of the cosmogonical hypothesis with its clear Copernican undertone in the Discorsi is especially remarkable in the light of the fact that the Discorsi were published after Galileo’s trial –a trial that was itself triggered by his engagement in Copernicanism.

5 A series of articles on Galileo’s cosmogony was published in the 1960s. The lively but short lived discussion revolved essentially around the two questions “why did Galileo attribute his hypothesis to Plato?” and “why was he so deeply committed to his hypothesis?”. The discussion can be reconstructed starting from Cohen, I. B., 1967.
computations possibly made by Galileo that would justify his claim of empirical adequacy. This situation changed when Stillman Drake in 1976 discovered that Galileo, in various calculations scattered over three folios of Ms. Gal 72, had used numbers taken from Kepler’s *Mysterium cosmographicum*. He successfully linked these calculations to Galileo’s work on cosmogony and offered a preliminary interpretation.

However, before turning to an interpretation of Galileo’s elaborations of his cosmogonical hypothesis, two theorems that are essential for their understanding—the law of fall, and the so-called *double distance rule*—will be discussed. What is needed, in particular, is the *law of fall* in its geometrical or *mean proportional* form which can be found in the *Discorsi* as the second corollary to proposition II on accelerated motion:

It is deduced, second, that if at the beginning of motion there are taken any two spaces whatever, run through in any [two] times, the times will be to each other as either of these two spaces is to the mean proportional space between the two given spaces.

The second theorem important for understanding Galileo’s work on cosmogony is the *double distance rule*. Even though never explicitly formulated as a theorem in the *Discorsi*, this rule was one of Galileo’s earliest and most important conceptual tools. The *double distance rule* states that a body whose motion is diverted into a uniform motion after fall through a certain distance will, in the time it took it to fall, traverse in uniform motion twice the distance fallen.

If Galileo’s cosmogonical hypothesis were correct it should be possible to derive essential features of the planetary system with the help of these two theorems. Indeed, given the point from which the divine creator drops the planets, their accelerated motion is determined by the law of fall, while properties of their uniform orbital motion can be inferred with the help of the *double distance rule*. But how exactly did Galileo test his cosmogonical hypothesis? Three folios containing material relevant to Galileo’s work on the cosmogonical hypothesis are readily identified by the appearance of the

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6 See Kepler, J., 1981, henceforth referred to as *Mysterium*. From a note of thanks sent back to Kepler we know that Galileo had received a copy of Kepler’s *Mysterium* in 1597. See Rosen, E., 1966 for an account of this first contact between Galileo and Kepler.

7 For Drake’s initial account see Drake, S., 1973. Drake later admitted a misreading of an abbreviation that, even though seemingly crucial to his interpretation, did not lead to its revision (Drake, S., 1978).

8 See Galilei, G., 1974, pp. 170-171. With A and B denoting the distances run through, and TA and TB the respective times of fall, the law of fall can (in modern notation) be expressed as \( TA : TB = A : mp(A|B) \). Here \( mp(A|B) \) specifies the mean proportional between the distances A and B, synonymous with the geometric mean of the two distances.

9 For a comprehensive account of the *double distance rule* and its role within the developing conceptual framework of Galileo’s science of motion see Damerow, P., et al., 1992.
number 10759, the revolution time of Saturn in days as given by Kepler in his *Mysterium*. As will be shown, two of these, folios 134 and 135, document Galileo’s first approach to testing his hypothesis. They contain calculations and diagrams referring to the same geometrical set-up and are hence interpreted here as belonging to the same approach.

On folio page 135 verso Galileo starts this first attempt by identifying a circle with a radius of 35 units and consequently a circumference of 220 units with Saturn’s orbit. According to Galileo’s cosmogonical model there must be a creation point from which Saturn was originally dropped and whose height has to be determined from the astronomical data found in Kepler’s book. The size of the orbit, fixed by Galileo’s assumption, together with the period of Saturn’s revolution fully determine the planet’s uniform motion along its orbit. As mentioned above, the double distance rule makes it possible to determine from an accelerated motion over a given space and time the distance traversed in the same time by a uniform motion resulting from a deflection of the accelerated motion. What is actually needed in this case, however, is a reversal of this procedure allowing to determine a distance of fall from a given uniform motion. It was therefore plausible for Galileo to assume that Saturn, since it covers in its uniform orbital motion a distance of 220 units during its revolution time of 10759 12/60 days, has in the preceding free fall covered in the same time half the distance, that is 110 units (Fig. 1).

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10 The orbital data used by Galileo in the cosmogonical calculations can be found in the last two chapters of Kepler’s *Mysterium*. A detailed discussion of the approach Kepler took on the question of planetary motion in the *Mysterium* can be omitted here, since it is essential only to Galileo’s second approach to the cosmogonical problem, in folio 146, which will not be discussed in this article. In the first approach to the problem, Galileo simply used the revolution periods of the planets he found in a table in chapter 20 of the *Mysterium*. The exact amount given for the period of Saturn in this table is 10,759 and twelve sixtieth days.

11 Folios 134 and 135 are physically connected and together form a larger sheet of paper. Both bear a double watermark of the type crown/crossbow and can hence with high probability be attributed to Galileo’s Paduan period. These two folios have been discussed in the literature. Stillman Drake dedicated a short paragraph to their discussion without linking their content directly to Galileo’s cosmogony. Drake does not deal with the content of folio 135 at all, while the content of the folio 134 is only touched upon briefly in a rather speculative paragraph. Following up on Drake’s work, Eric Meyer realized that the calculations on these two pages represent an early approach by Galileo to testing his cosmogonical hypothesis but failed to give a satisfactory interpretation because of a puzzle he had encountered but did not solve, see Meyer, E., 1989.

12 This choice of a radius for Saturn’s orbit merely fixes the unit in which the cosmic distances are to be measured. Since Galileo uses the numerical value 3 1/7 for pi, a diameter of 7 naturally offers itself if one aims to arrive at an even number for the circumference. Scaled by 10 this value represents a convenient choice for subsequent calculations.
As a next step Galileo tried to test the empirical adequacy of his hypothesis by determining the motions of the inner planets resulting from a fall from the creation point whose position he had determined from Saturn’s motion. How can the motion of one of the inner planets be determined? If the size of the orbit is assumed to be known, the time it takes an inner planet to fall from the given creation point to its orbit can be found with the help of the law of fall. After being diverted into its orbit, the inner planet will, according to the double distance rule, traverse twice the distance fallen in the time of the motion of fall. This distance will no longer, in contrast to the case of Saturn, coincide with half the length of the orbit’s circumference.

Fig. 2: Determination of the motion of an inner planet.
Accordingly also the revolution time of the planet will no longer coincide with the time of fall but it can be calculated from the ratio between double the distance of fall and the length of the orbit. In short, the determination of the period for an inner planet from its orbital geometry involves three steps, first the application of the law of fall to the distance between the creation point and the orbit, then an application of the double distance rule to the relation between the accelerated motion of fall and the uniform orbital motion, and finally an application of a basic theorem on uniform motion yielding the period of revolution (Fig. 2).

If the proportions yielded by these three steps are combined one arrives, by employing basic proportional theory, at the proportion shown in Figure 3, henceforth referred to as the first cosmogonical proportion.\(^{13}\)

![Table 1: First cosmogonical proportion used on folio page 135 verso.](image)

As a matter of fact, the first cosmogonical proportion provides Galileo with two alternatives for testing whether the actual motion of the planets is in accordance with the cosmogonical hypothesis. It can either be exploited to determine the planetary geometries from given periods or it can be used to determine the periods from a given planetary geometry. If the hypothesis were adequate, the results of both approaches would coincide and yield the empirical data. Given the fact that Galileo made these calculations at a time when the periods of the planets had been observed with pretty high accuracy but little was known about the actual planetary geometry, he reasonably

\[^{13}\] In this and in the following footnotes capital letters represent a quantity that is a time or a distance. A subscripted letter further specifies the quantity as pertaining to one of the planets, where a subscripted S denotes Saturn, a subscripted J Jupiter. Accordingly Ts and Tj represent the times of fall of Saturn and Jupiter over the respective distances of their orbits from the creation point, Fs and Fj. As a consequence of Galileo’s earlier choice Ts is identical to the period of Saturn Ps. Then according to the law of fall Tj : Ts = mp(Fs|Fj) : Fs. According to the double distance rule Jupiter after being diverted in its orbit will cover 2Fj in the time Tj. Then since in uniform motion the times are in the same proportion as the distances covered Pj : Tj = Oj : 2Fj, where Oj denotes the size of Jupiter’s orbit. Hence by compounding proportions Pj : Ps = (mp(Fs|Fj) : Fs) x (Oj : 2Fj) which according to proportional theory is equivalent to Pj : Fs = Oj : 2mp(Fs|Fj), the first cosmogonical proportion.
chose the first approach, namely to determine the planetary geometry from the periods. However, this approach was hampered by a difficulty resulting from the fact that he could not resolve the first cosmogonical proportion for one characteristic magnitude of the orbital geometry but rather had to rely on a laborious iteration procedure in order to determine such a magnitude from the given periods and the given position of the creation point.¹⁴

On folio page 135 verso the first cosmogonical proportion is exploited to determine Jupiter’s planetary geometry from the periods of the planets given by Kepler. From a first guess of Jupiter’s planetary geometry, a period for Jupiter is calculated. According to the result the geometry is then varied until the period resulting for Jupiter is within an acceptable limit identical to the one given by Kepler.¹⁵

One would expect that Galileo went through this procedure four times to determine the orbital geometries of the other inner planets Mars, Earth, Venus and Mercury. Indeed on folio page 134 verso we find the calculations for Mars. But only at first glance do the calculations follow the same scheme. A closer look reveals that in place of the first cosmogonical proportion he had used on folio page 135 verso, he uses here a modified proportion, in the following called the second cosmogonical proportion, to determine the orbital geometry of Mars from its period (Fig. 4).

¹⁴ Doubts concerning the precision of the sizes of the Copernican orbits could have been furthered by Galileo’s reading of chapter 18 of the Mysterium, in which Kepler elaborates on the disagreement between his theoretical values and the sizes of the Copernican orbits as well as on the precision of astronomy in general. Yet Galileo with the mathematical tools available to him could not exploit the first cosmogonical proportion to determine Oj from the given ratio of the revolution times directly. A complicated relation relates the size of Jupiter’s orbit to the mean proportional of the distances of Saturn’s and Jupiter’s orbits from the creation point. Consequently Galileo can only determine Oj from a given mean proportional mp(Fs|Fj) in three steps. First Fj is determined from Fs and the mean proportional mp(Fs|Fj) according to Fj = mp(Fs|Fj)²: Fs. In the next step the radius of Jupiter’s orbit Rj is calculated as the difference between the distance of the creation point from the center and the distance fallen by Jupiter, Fj. The size of Jupiter’s orbit is calculated from knowledge of the size of its radius according to Oj = 2 x 3 1/7 x Rj. Finally in a last step the resulting period of Jupiter is calculated according to Pj = Oj : 2mp(Fs|Fs) x Ps, in conformity with the first cosmogonical proportion.

¹⁵ On folio page 135 verso Galileo applies the procedure to determine Jupiter’s period Pj from a given mean proportional mp(Fs|Fj) described in the preceding footnote a full five times. The subsequent choices for the numerical value of the mean proportional, 120, 116, 118, 119 and 119 1/4 result in the following periods for Jupiter, 59[67], 6608, [528], 45[xx] 44[59]. Digits not written down by Galileo are given in square brackets. The final period calculated for Jupiter has to be compared to the actual period of 4333 days.
First cosmogonical proportion used on folio page 135 verso

\[
\frac{\text{revolution time inner planet}}{\text{revolution time Saturn}} = \frac{\text{orbit of inner planet}}{2 \times \text{mean proportional of inner planet’s and Saturn’s fall}}
\]

Second cosmogonical proportion used on folio page 134 verso

\[
\frac{\text{revolution time inner planet}}{\text{revolution time Saturn}} = \frac{\text{orbit of inner planet}}{2 \times \text{fall of inner planet}}
\]

Fig. 4: Conflicting proportions used in the calculations for Jupiter and Mars.

As it turns out, the second cosmogonical proportion is, from Galileo’s perspective, just as justified as is the first. Instead of just using the law of fall and the double distance rule, the method of calculation underlying this second proportion is based on a principle of his theory of motion that is incorrect according to classical mechanics: the proportionality between the distance of fall and the “degree of speed” it reaches at the end. From a famous letter written to Sarpi in October 1604, we know that Galileo at that time adhered to this principle of fall on which he hoped to found his new science.\(^{16}\) The use in the cosmogonical calculations of the principle mentioned in this letter suggests to date the calculations previously discussed to a time period whose boundaries are determined by the fact that Galileo is already using the law of fall while still employing his early erroneous principle of fall.\(^{17}\)

If this principle is applied to the case under consideration, the necessary calculations may be simplified a bit, which possibly constitutes the reason why Galileo adopted this second method –which from his point of view must have been equivalent to the first– to the determination of the orbit of Mars. According to Galileo’s erroneous principle, the degrees of speed of Saturn and an inner planet upon reaching their respective orbits are to each other in the same ratio as the distances fallen. A proposition on uniform motion then allows one to determine the ratio of times of the two uniform planetary motions covering orbits of different sizes with the different speeds resulting from their fall. Since this reasoning yields exactly the second cosmogonical proportion used on folio page 134 verso, it becomes probable

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\(^{16}\) See the letter to Paolo Sarpi, October 16, 1604, in Galilei, G., 1890-1909, X: 115f.

\(^{17}\) Such a dating is in accordance with other proposed evidence for dating, especially with the conjecture that a letter by Edmund Bruce written to Kepler in 1602 refers to Galileo’s work on cosmogony. It is however less speculative and at the same time has the prospect of being refined as we learn more about the development of Galileo’s science of motion.
that Galileo’s principle mentioned in the letter to Sarpi constitutes indeed the basis for this calculation.\footnote{With GVs and GVm representing the degrees of speed of Saturn and Mars upon reaching their orbits, the principle that the degrees of speed grow in proportion to the distances fallen can be expressed as GVs : GVm = Fs : Fm. Together with Galileo’s common assumption that in the deflection of an accelerated motion into a uniform motion the ratio of the degrees of speed is preserved in the ratio of the speeds of the resulting uniform motion, the proportion also holds for the orbital speeds Vs : Vm = Fs : Fm. Since in uniform motion with unequal speeds, the ratio of the times is compounded from the ratio of spaces and from the inverse ratio of speeds and the size of Saturn’s orbit amounts to twice the distance fallen by Saturn the following proportion holds Pm : Ps = Om : 2 Fm, the second cosmogonical proportion. Just as before this proportion is exploited to determine the size respectively the radius of Mars’s orbit.
}

Ergo both cosmogonical proportions, even though contradictory from the perspective of classical physics, turn out to be justified within the framework of Galileo’s science of motion in a particular stage of its development. But did Galileo realize this internal contradiction in his approach to the problem? At least not immediately, because the diagram he drew for the three orbits of Saturn, Jupiter, and Mars on the reverse of folio 134 contains a further orbit showing that Galileo had carried out computations also for the Earth without in any way revising his earlier results.\footnote{Meyer has pointed out that the radii of the three outermost of the four concentric circles on folio page 134 recto are in the same ratio as the radii of Saturn Jupiter and Mars as resulting from Galileo’s considerations. Meyer further claims that the scattered calculations on this page represent attempts to reduce the calculated ratios to a series of simple ratios. The inner circle represents Earth’s orbit drawn to scale with a ratio resulting from a calculation similar to the ones discussed. Whether the first cosmogonical proportion or the second cosmogonical proportion have been used in the determination of Earth’s radius cannot be established with certainty since both approaches involve a break off of the calculation procedure when a satisfactory agreement with Earth’s actual period of 365 days is reached.}

There is no indication whatsoever, whether, for the time being, Galileo judged his cosmogonical calculations to be successful or not. What had been accomplished was, in any case, the calculation of orbits so that the planets would move in these orbits with the observed orbital periods. However, a comparison of these calculated orbits with the sizes of the Copernican orbits used in Kepler’s book shows crude deviations.

While the implications of Galileo’s calculations for the understanding of cosmogony may have been doubtful, these calculations probably had a profound impact on Galileo’s science of motion. Folio page 134 recto contains a diagram representing a motion of fall, interrupted at two points to generate a uniform horizontal motion. This diagram represents, in a nutshell, the essential mechanism of Galileo’s calculations based on three fundamental ingredients, the law of fall, the double distance rule, and the erroneous proportionality between the degrees of speed and the distances of fall. As is known from the analysis of other folio pages of Ms. Gal. 72, Galileo eventually realized and elaborated the internal contradiction between his erroneous principle and the law of fall on the basis of considering exactly the
Indeed, Galileo’s refutation of his erroneous principle of fall has precisely the same structure as his cosmogonical hypothesis, which also involves a comparison between different uniform motions generated by a motion of fall, and may well have been triggered by the work on this hypothesis. In other words, Galileo’s elaboration of his theory of motion to include cosmogony may have had far-reaching repercussions on its very foundations. The insight into the contradiction between the law of fall and the early erroneous principle led to a conceptual revision of the foundations of his theory of motion, requiring in particular a shift to a new principle of fall, according to which the degrees of speed increase with the times fallen.

When Galileo returned to the problem of cosmogony for a second time after the revision of his theory, he consequently had to modify his approach. His later approach, which will not be discussed here, also avoided other shortcomings of his first attempt. In fact, he now not only used the correct proportionality between the degrees of speed and time, but also managed to reproduce with his model the planetary data, orbits as well as periods, for two planets, and he developed a more adequate method of calculation. Even though Galileo again did not achieve full correspondence between Kepler’s astronomical data and the results of the calculations based on his own hypothesis, this second approach represented a far more adequate appropriation of the problem to his science of motion. The relative success of the second approach may well have constituted the background of Galileo’s self-assured public statements on the cosmogonical hypothesis.

Historians of science have extensively discussed the interrelations between Galileo’s mechanical thinking and his astronomical thinking. They have in particular tried to answer the question of whether it was a fully developed Copernican program that shaped Galileo’s science of motion or whether it was only his theory of motion that led to the development of such a program. The example presented in this paper implies, however, that the question is posed too narrowly. The interpretation given shows that Galileo tried to integrate his new science of motion with Copernican con-

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20 These considerations are documented on folio page 152 recto. For a comprehensive interpretation see Damerow, P., et al., 1992, pp. 185-194.

21 As discussed above, Galileo, in his determination of the position of the creation point, had assumed that Saturn had fallen from the creation point to its orbit in exactly its revolution period. While this condition may have appeared natural in view of the double distance rule, it actually turned out to be too restrictive. In fact, it fixes the ratio between distances traversed and times consumed in free fall, in modern terms it fixes a constant of acceleration. It is, on the other hand, exactly the freedom in this choice of the relation between distance fallen and time consumed that allows Galileo in his second attempt at the problem to determine the creation point from the orbital data of two planets such that vice versa fall from this creation point yields the correct orbital data for these two planets. I plan to include a more detailed study of Galileo’s work on cosmogony including his second attempt documented on folio 146 in my dissertation project on the development of Galileo’s science of motion.

22 See for example McMullin, E., 1967.
cepts already at an early stage, that is at a time when neither his science of motion nor his Copernican position were fully developed. As a matter of fact, in Galileo’s time, the omnipresence of Aristotelian natural philosophy and the astronomical issues related to it created boundary conditions that no attempt at a new science of motion could ignore.\textsuperscript{23} Galileo’s early attempt of an integration of his new science of motion with cosmological issues hence does not bear the characteristics of a strategically planned step, but has rather to be interpreted as an unavoidable encounter that affected both, Galileo’s understanding of cosmology and his theory of motion.

References


\textsuperscript{23} For a more detailed account of this view of the interrelations of Galileo’s mechanical and astronomical thinking see the contribution by Büttner, Damerow and Renn in this volume.
Dialogo terzo

SIMP. Sia questo segnato A il luogo del globo terrestre.

SALV. Bene sia. Se secondariamente, che voi sapete benissimo, che cosi terras non è dentro al corpo solare, né meno a quello contiguo, ma per certo spazio distante, e però assegnate al Sole quel altro luogo più o meno remoto della terra a vostro beneplausio, e questo ancora contraffingate.

SIMP. Eco fatto: Sia il luogo del corpo solare questo segnato O.