Isaac Barrow on God, Geometry, and the Nature of Space

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INTRODUCTION

Historians used to portray Isaac Barrow (1630-1677) as an obscure figure in Newton's background. He was a Janus-faced figure mainly of interest —one of his biographers once famously said— because he was "a link between the old and the new philosophy," the old being his and the new Newton's.[1] This theme became commonplace in dealing with Barrow. For decades, and with the exception of Barrow's role in the philosophical articulation of the idea of absolute time, Barrow's contributions were found to be hopelessly out of tune with contemporary modern intellectual tendencies. Consequently, attempts to trace any kind of substantial influence from Barrow on Newton always found an unsympathetic reception.[2] The most important attempt to

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[1] P.H. Osmond, *Isaac Barrow, His Life and Times* (London: Soc. for the Promotion of Christian Knowledge, 1944), p. 1.

[2] Barrow's articulation of the notion of absolute time is particularly well analyzed in E.A. Burtt's old but still useful *The Metaphysical Foundations of Modern Physical Science* (Atlantic Highlands, N.J.: Humanities Press, 1989, first ed. 1924), p. 150-61. On Barrow's notion of space, see E. Grant, *Much ado about nothing. Theories of space and vacuum from the Middle Ages to the Scientific Revolution* (Cambridge: Cambridge University Press, 1981), p. 236-238. See also E.W. Strong, 'Barrow and Newton', *Journal of the History of Philosophy* (1970), *8*, 155-72; A. R. Hall, *Henry More. Magic, Religion and Experiment* (Oxford: Basil Blackwell, 1990), p. 199ff. Notice that M. Jammer's influential *Concepts of Space* (New York: Dover Publications, 1993, first ed. 1954), wrongly claims that Barrow "promulgated [Henry] More's ideas [on space] in mathematized form in his *Mathematical Lectures*. In Barrow's geometry, space is the expression of divine omnipresence" (*ibid*., p. 110-1).

credit Barrow with fresh methodological ideas that might have influenced Newton came from Robert Kargon, but his suggestion was completely overlooked.[3]

No doubt because of the loss of most of Barrow's scientific manuscripts, of the scarcity of documentary evidence regarding his life, but also because of the distorting effect induced by the magnification of Newton's figure, it has proved difficult to focus on Barrow's own contributions to mathematics and natural philosophy. Fortunately enough, in recent years M. Feingold’s book, with excellent studies on Barrow's life, his academic context, and his contributions to mathematics and optics, has done much to make of Barrow a very interesting figure in his own right.[4] Barrow's best-known scientific works are three sets of “lectures” he prepared in fulfillment of his duties as Lucasian professor of mathematics betwen 1664 and 1669.[5] The *Geometrical Lectures* and the *Optical Lectures* have been recently the object of careful study. The third set, the *Mathematical Lectures,* include a long, substantial philosophical analysis of the object and method of the mathematical sciences, apparently delivered as public lectures in three consecutive years from 1664 to 1666. Hitherto largely neglected, this facet of his natural philosophy shows him to be a well-informed, deep, original thinker. The

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[3] R. Kargon, *Atomism in England from Hariot to Newton* (Oxford: Clarendon Press, 1966), p. 106-117; 'Newton, Barrow, and the Hypothetical Physics', *Centaurus*, 1965, *11*, 46-56.

[4] *Before Newton. The life and times of Isaac Barrow*, M. Feingold, ed. (Cambridge: Cambridge University Press, 1990).

[5] Published by W. Whewell under the title of *Mathematical Works* (Cambridge, 1860). They include the *Mathematical Lectures*, the *Optical Lectures*, and the *Geometrical Lectures*. Unless otherwise stated, English quotations from the *Mathematical Lectures* (first printed in Latin in 1683, reprinted in 1684 and 1685) will come from John Kirkby's translation, *The Usefulness of Mathematical Learning explained and demonstrated*, London, 1734. English quotations from the *Optical Lectures* (first printed in Latin in

present paper aims to explore Barrow’s notion of space, particularly in connection with the philosophical and theological debates raging among natural philosophers (obviously including Hobbes) in the early years of the Restoration.

Barrow's attitude towards natural philosophy underwent a marked evolution. When a young Cambridge graduate around 1650, he had an earnest interest in medicine. Then, apparently under the influence of Henry More, he become engrossed in natural philosophy and produced a thesis on the insufficiency of the Cartesian hypothesis. During the Civil War, after his failure to secure a Greek professorship under Cromwell, he spent four years on an extra-long Continental tour that took him to Turkey, settling for one year in Istanbul. Back in England and having won the Lucasian professorship of Mathematics, his interests turned to pure mathematics and geometrical optics, for him the perfect example for the strong program of mathematization of natural philosophy that he articulated in his *Mathematical Lectures*. Finally, in 1669, the eminent Lucasian professor of Mathematics became Chaplain of his Majesty and eminent Restoration divine, and turned to theology.

In going over Barrow's biography, particularly when reading contemporary accounts of his life and character, it is hard to avoid the impression that he grew ever more skeptical about the fruitfulness and worth of natural philosophy. It is my impression that, as his skepticism grew, he first took refuge in geometrical optics and mathematics, and reivindicated a strongly mathematized natural philosophy.[6] Then, in the last years of his short life, Barrow eventually abandoned mathematics and became a

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1669) will come from *Isaac Barrow's Optical Lectures*, H.C. Fay trans., A.G. Bennett, D.F. Edgar eds. (London: The Worshipful Company of Spectacle Makers, 1987).

prolific author of sermons and religious writings. First published shortly after his death, by none other than John Tillotson (1630-94), later archbishop of Canterbury, his theological works occupy four thick folio volumes with over two thousand pages in all.[7] Overloaded with involved and sophisticated philosophical arguments, his sermons may have not been very popular--indeed there is some evidence that they were not. On the other hand, they were highly regarded by scholars as well as by the Anglican hierarchy.[8] It is on certain of his sermons, as well as on the philosophical discussions contained in the *Mathematical Lectures* that my account of Barrow's philosophy of space will rest.[9]

Let me stress first of all that not a minor feature of Barrow's theology and its accompanying program for the mathematization of natural philosophy is its being explicitly grounded on philosophical and theological principles that reappear prominently in apologies of Newton's work thirty or forty years later. Certainly, in rhetoric as well as intention Barrow's sermons often ring like turn-of-the-eighteenth-

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[6] See the 'Life' which prefaces his *Works*, n. (2), and A. Hill, 'Some Account of the Life of Dr. Isaac Barrow', in Napier ed., *Theological Works*, n. (6), I, xxxvii-liv. For recent biographical studies, see the contributions of M. Feingold and J. Gascogine in Feingold, *Before Newton*, n. (4).

[7] See *Works*, n. (2). A nineteenth-century edition by A. Napier exists: *The Theological Works of Isaac Barrow* (9 vols.), Cambridge, 1859.

[8] J. Gascoigne, 'Isaac Barrow's academic milieu: Interregnum and Restoration Cambridge', in *Before Newton*, op. cit (4), 250-90, p. 279-81.

[9] Both because of Barrow's statements concerning the origins of this text, and because of its content and style there can be little doubt that the *Mathematical Lectures* were actually read in the classroom under the form they have reach us, or in a form very similar to it--which is something that cannot be said of the 1670 *Geometrical Lectures*. For a very good discussion of aspects of the *Mathematical Lectures* not

century Boyle lectures. Important though it is, there is today no time for me to dwell in this meaningful link between the young Newton's intellectual context and the peculiar response his contributions met in England from the 1690s on —peculiar of course by opposition to the Continental response. It will not be amiss, however, to mention that by 1670 or thereabouts Barrow was delivering sermons with such self-explanatory titles as "The Being of God Proved from the Frame of the World" and "The Being of God Proved from the Frame of Human Nature". [10] They not only fully display the design argument, but also contain metaphors and arguments that will turn particularly influential in Newtonian commentary of later years, of which I shall provide just two examples.

After having set forth the notion that the complexity of the world precludes "chance" from having any role in its formation, he then suggests that God actually, directly, and continuously keeps the world ("this great Machine") together, and maintains it so that it runs smoothly and without deteriorating:

> So that six thousand years together hath this great *Machine* stood, always one and the same, unimpaired in its beauty, unworn in its parts, unwearied, and undisturbed in its motions.[11]

My second example concerns the relation of God with his creation, which Barrow solely explains in terms of the analogy —so important to Newton, as we know— between the human soul and the body [stress is found in the original]:

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dealt with in this paper, see M.S. Mahoney, 'Barrow's mathematics: between ancients and moderns', in Feingold ed., *Before Newton*, n. (4), 179-249.

[10] *The Works of Isaac Barrow published by ... Dr Tillotson*, 4 vols., (London, 1683-1687), II, 86-99, 100-112.

[11] Barrow, *Works* II, p. 98.

> In the substance of man's soul, in its union with things corporeal ... we may observe diverse ... resemblances [between the human soul and God]. ...
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> As God by His presence and influence doth ... *contain, and keep together the whole frame of things*; so that He withdrawing them, it would fall of itself into corruption and ruine; So doth the soul by its union and secret energy upon the body connect the parts of its body, and preserve it from dissolution, which presently, being removed, do follow.
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> As He, in a manner beyond our conception ... doth co-exist with, penetrateth, and passeth through all things; So is she, in a manner also unconceivable, every where present within her bounds, and penetrates all the dimensions of her little world.

And then Barrow adds words that remind us of what Newton had to say about the presence of God in absolute space —be it His *sensorium* or not:

> He incomprehensibly by a word of his mind, or by a mere act of will doth move the whole frame or any part of nature.[12]

THE *MATHEMATICAL LECTURES* (1664-1666)

Barrow's *Mathematical Lectures* contain three series of lectures, one per year. Thematically, they have a clear structure. The eight lectures of the first year mostly deal with the nature and status of the mathematical sciences — whether mathematics is a science, how many mathematical sciences are there, the nature of mathematical demonstration, the nature of definitions and first principles, what is the relationship between algebra and geometry, and so on. The seven lectures of the second year deal with mathematical magnitude, its nature, its mathematical properties, its measure, and how it grounds the notion of number. Finally, the eight lectures of the third year focus

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[12] Barrow, *Works* II, p. 105.

on ratio and proportionality, a hotly debated subject among 17th-century mathematicians of all countries and conditions.

Ostensibly, Barrow's *Mathematical Lectures* were addressed to Cambridge undergrads with little knowledge of and less interest in mathematics, to serve them as a non-technical introduction to the basics of mathematics and of what one would nowadays call the philosophy of mathematics—words Barrow himself never used. At close inspection, however, these *Lectures* reveal theologically informed discussions of some of the hottest and most controversial topics involving Cartesians, atomists, and philosophers generally, whether experimental, mechanical, or otherwise. This is particularly true of the lectures delivered in the first two years, where Barrow brings forth his notion of God to discuss subjects such as whether necessary demonstrations are at all possible, what kind of causality can be introduced into scientific demonstrations, how God takes care of his creation, whether other worlds can possibly exist, and then how our mathematical knowledge would apply to them, whether all extension is matter, whether matter might be necessarily infinite, whether space exists, and then what its nature might be, and so on. The *Lectures*, therefore, were Barrow's instrument to teach undergraduates the need to link knowledge of God to knowledge of nature and mathematics. He taught them by example the principle —commonly assumed in 17th-century Europe, particularly in Protestant countries— that Newton made explicit in one fragment of his (never finished) history of science: "knowledge of God's works thrived in those epochs in which there was a true conception of the Deity; and conversely".[13]

[13] F.E. Manuel, *The Religion of Isaac Newton* (Oxford: Clarendon Press, 1974), p. 42.

BARROW ON SPACE

In one of the most famous and most praised foundational lies ever written in science, Newton in his opening scholium to the *Principia* famously said, “I do not define time, space, place, and motion as being [words] well-known to all.”[14] Newton notwithstanding, we know that ‘time’ and ‘space’ stand out among the handful of words 17th-century philosophers of all persuasions loved to disagree upon. In fact, as is well known, the notion of space had been hotly debated almost continuously from classical times up through the 17th century. While I cannot and I shall not offer here a full, comprehensive review of 17th-century discussions on the nature of space, I will still briefly mention the main points and arguments under discussion so that we may appreciate the originality of Barrow’s stance.

In one sense, Newton was right. By the second half of the 17th century, space was an idea whose time had come. Efforts to clarify it were many and arguments for the existence of space as an entity separate from the things that occupy it came from very different quarters. Moreover, while it proved extremly difficult to agree on a metaphysical definition of space, yet most thinkers agreed in endowing “space” with similar sets of properties. As Barrow put it in his *Mathematical Lectures,* the idea of space is received by almost everyone, and the “common consent of mankind” supports the idea —he would not discuss whether it is either innate or otherwise acquired— that space is something different from the things that occupy it. Barrow, who believes the notion of space is “engraven” (*insculpto*) in the imaginations (*phantasia*) of almost all

[14] I. Newton, *Mathematical Principles of Natural Philosophy*, 2 vols., A. Motte, trans., F. Cajori, rev. (Berkeley: University of California Press, 1962), I, p. 6.

mortals, provides a vivid characterization of this idea of space that was "well-known to all":

> The vulgar are used to imagine that there is some common substratum to all things that is infinitely extended (*infinite distendatur*) and circumscribed by no limits; that is perfectly penetrable and most easily admits everything within itself, not resisting the entrance of anything; that receives the successions of moveable bodies, determines the velocities of motions, and measures the distances of things; that is immoveably fixed, … nor can possibly be any where transferred from where it is; lastly, that is a receptacle of immense capacity, ...[15]

Assuming that the wide "consensus of the imagination" over this "vulgar" idea of space cannot be dismissed, Barrow argues for the existence of space as something distinct from the things that occupy it and counters the objections commonly raised against it. The main arguments against the existence of space were theological, ontological, and logical. First of all, if space was supposed to be an unproduced receptacle, an immaterial substratum, eternal, infinitely extended, a precondition for the location of everything else, existing in itself independently of everything else, then space would not only be uncreated but also independent from the divine will and providence. For many 17th-century thinkers, including Barrow, the existence of anything endowed with such properties was contrary to reason and religion.[16] Interestingly, this notion of space, which was closest to Patrizi's, Gassendi's, and to 17th-century atomists generally, was not the one that Barrow found most dangerous. I think the reason is to be found in that its attendant theological dangers could easily be

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[15] Barrow, *Usefulness,* p. 165.

[16] Barrow, *Usefulness*, p. 164.

sidestepped either via Henry More's conflation of space with some divine attributes, or via Barrow's own idea of space (more about this below).

A second kind of argument against immaterial space comes from the difficulty of ontologically distinguishing space from, magnitude. Since the main and essential properties of space are extension and unlimited capacity, which are also essential properties of magnitudes in general, why should they be distinguished? Barrow took eagerly the task of criticizing this line of thought, most fully and powerfully developped in the 17th century by Descartes. As we shall see, Barrow's main attacks took the form of long, careful, and detailed discussions of the dangers of countenancing the existence of infinitely extended matter.

A third family of arguments pointed to the impossibly difficult problem posed by the conceptualization of space within the categories then available. If space is something really existing "out there", what is it, a substance or an accident? Space could not be a *material* substance, since it obviously lacks matter. Could it then be an *immaterial* substance? According to Henry More it could and it was. According to everybody else, however, it could not, since immaterial substances were assimilated to spiritual agents, and therefore essentailly indivisible, while space is as much divisible as any corporeal thing. Then, was space an accident? If it were, which substance would space be an accident of? Space could not be an accident because it was called on to be that permanent reference that depends on no particular substance and remains when all substances are removed from it.[17] Barrow (as Gassendi and Hobbes also did) recognized the dicotomy substance-or-accident as a serious challenge for anyone willing to grant independent reality to the notion of space. In Barrow's presentation of

[17] For the presentation of this argument in renaissance Aristotelanism, see ; Grant, *Much Ado*, p. 157-8.

this difficulty, he dismissed that space could be a substance but merely claiming that none of those who advocate the independent existence of space grant it "the dignity of a substance". On the other hand, he argued that space cannot be an accident either, because "[space] is extrinsecal to all substance, and it is not carried away [by any substance], but remains although [any substance] be taken away, and it depends upon no other thing."[18]

For obvious reasons, this logical conundrum was not a serious problem for Descartes, nor for More. Gassendi, however, could not easily solve it. His radical solution was to construct space and time as fundamental, primary, out-the-ordinary categories that do not fall "within the general division of Being, or Things, between substance and accident."[19] Hobbes's own radical solution was to redefine the notion of accident and turn space into an accident of the human imagination. As it was the case with Descartes's notion of space, Hobbes's views deeply concerned Barrow, who took his time to criticize them while offering his own way out of the impasse.

Barrow grounds his criticism of Descartes in three premises or principles that he derives directly from his understanding of God. Barrow's first thesis is that there is nothing in matter itself, nor in God's relation to matter that makes necessary the existence of infinite matter. The authority of quotations from the Bible, is adduced in support of Barrow's thesis, but to the escriptural quotations he adds his voluntaristic understanding of God, according to which He is a "perfectly free and independent

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[18] Barrow, *Usefulness*, p. 164-5.

[19] Gassendi, *Syntagma*, I, p. 179 – 182 (quotation on p. 179); see O.R. Bloch, *La philosophie de Gassendi* (La haye: Martinus Nijhoff, 1971), p. 172-181; Grant, *Much Ado*, p. 209-210.

agent", under no necessity whatsoever to attribute infinity to matter.[20] Since Barrow assumes the extension of space necessarily unlimited, Descartes's univers with its necessarily indefinite material extension becomes problematic. Actually, Barrow uses the distinction between space and matter as a crucial argument to establish the existence of space. By comparing the essential infinity of God with the finite extension of matter (or its contingently indefinite extension), Barrow concludes that there must be something or somewhere beyond matter where He may subsist. Otherwise, "our imagination [*imaginatio*] could conceive a place where He is not, and therefore somehow transcend the manner [*modum*] of divine existence, and so we could not apprehend or know God's inmensity."[21]

Barrow's second principle is that God can "according to what pleases him" increase or diminish the quantity of matter, and in particular, that He could annihilate all matter. In support of this assumption Barrow brings in God's omnipotence and that reason does not contradict but rather supports it.[22] Barrow's third premise, closely connected to the second one, is that God may preserve things as they are in a given moment and place, regardless of what happens to all matter inside or outside them. As we shall see presently, Barrow has in mind the surface of spheres or parallelepipeds, or the walls of any room, for they appear in a famous argument between Descartes and Henry More above the nature of space. Again, this assumption rests on the voluntartistic understanding of God that allows us to assume that He may act in any way that does not involve a contradiction.

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[20] Barrow, *Usefulness*, p. 166.

[21] Barrow, *Lectiones mathematicae*, p. 154.

[22] Barrow, *Lectiones mathematicae*, p. 153-4.

These two principles play a key role in a series of arguments where Barrow establishes the existence of space by elaborating the basic idea of the so-called "annihilation thought experiments". The trick of imagining that God annihilates everything from a given room (historically, the argument first appeared with the annihilation of everything within the lunar sphere) has a long pedigree reaching back at least to Roger Bacon, Bonaventure, and John Buridan. The medieval arguments mostly hinged around the properties and existence of the void (usually, extracosmic void). The annihilation thought experiment became relevant again in Jesuit and renaissance scholastic discussions of imaginary space.[23] In the seventeenth century the argument was alive and kicking, with stellar performances in Descartes's, Henry More's, Hobbes's, and Barrow's writings. In Barrow's case, he was probably responding to the arguments crossed between More and Descartes in his famous exchange of letters of 1648, republished by More just months before Barrow was lecturing in Cambridge about the nature of space, in 1662.[24]

As is well known, in his 1644 *Principles of Philosophy* Descartes used an annihilation thought experiment to support one of his key metaphysical tenets, that "there can be no extension that is extension of nothing." In Book II of the *Principles*, one of his main arguments against the existence of void, ie., space devoid of any material substance, was that if God emptied a vase of air, some other material must enter it or else its sides would collapse:

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[23] C. Leijenhorst, "Jesuit concepts of *spatium imaginarium* and Thomas Hobbes's doctrine of space", *Early science and medicine* **1** (1996): 355-380; Grant, *Much Ado*, p. 152-174.

[24] H. More, *Collection of severall philosophical writings* (London, 1662); the correspondence is more easily found in R. Descartes, *Oeuvres*, 12 vols., C. Adam, J. Tannery, eds. (Paris, 1903), V.

> It may be asked what would happen if God removed all the body contained in a vessel, and allowed no other body to come and take the place of what was removed. The answer must be that in that case the sides of the vessel would *ipso facto* be in contact; for when there is nothing between two bodies, they must necessarily touch each other. It is manifestly contradictory for them to be apart, or to have a distance between them, while at the same time the distance is nothing; for any distance is an aspect (*modus*) of extension, and thus cannot exist without an extended substance.[25]

In correspondence with Descartes, Henry More countered that the vase might remain empty of all material substance because the divine extension would fill the vase and if necessary hold its sides in place.[26] More represented space as a prime example of immaterial extended substance, a crucial notion in his metaphysics. In his mature works, More comes close to identify space with a facet of God's nature, turning it into what provides to our weak intellect a shadow or vague representation of the nature of the continuous divine omnipresence.[27] As we shall see, Barrow did not need to turn space into a subtance to make it something real, and although Barrow's God plays a role in his conceptualization of space, yet Barrow's answer to the Cartesian challenge was phrased in logical and physical terms. In Barrow's thought experiment he assumes the matter between the two concentric material spherical surfaces annihilated without the shapes and sizes of the spheres being in any way modified —something obviously in God's power, as argued previously. In an interesting reversal of Descartes's

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[25] *Descartes philosophical writings*, trans. and ed. by E. Anscombe, P.T. Geach (Indianapolis: Bobbs-Merrill, 1982), p. 206.

[26] Descartes, *Oeuvres*, X, p. 184.

conclusion, Barrow concludes that something, space, must remain between the spherical surfaces, otherwise they "will meet each other" (*sibi coincident*). Now, according to Barrow this is patently absurd because, given their different sizes and positions, all kind of geometrical paradoxes would follow.[28]

To make clear the absurdity of assuming that the sides of a vessel would necessary come to touch each other in all their points as a consequence of the removal of the matter they contain, Barrow connects this idea to motion. Barrow argues that, if no space or immaterial substratum existed underlying material objects, the removal of any of them —let us call it C—would imply an instantaneous, inevitable, self-activating contiguity of the material objects surrounding C. Notice that they would become contiguous or would come together without actually moving, and "without the application of any force", just as an inevitable result of there no longer being anything to separate them. Then, a shocking and objectionable asymmetry in nature would follow. Bodies A and B, surrounding C, would become contiguous of their own accord, without any other force than the one required to remove C. But we know that if A and B are contiguous, we need a strong force to separate them so as to be able next to move C in between them. If we cannot conceive bodies merely separated but space, then:

> "it can scarce be understood … why [bodies] should meet together of their own accord but be separated unwillingly: what the cause should be why they lose their former state without further ado and with no motion

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[27] A. Jacob, *Henry More's manual of metaphysics. A translation of the* *Enchiridium metaphysicum* *(1679)*, 2 vols. (Hildesheim: Georg Olms, 1995), ch. 8, I, p. 54-61.

[28] Barrow, *Lectiones mathematicae*, p. 155.

> but are only hardly restored to it by vehement labor [*nisu*] and much motion."[29]

Of course, Barrow's conclusion is that such an asymmetry cannot exist. This is why previously separated bodies do not become contiguous by the mere removal of what lies in between, and this is also the reason why "if a vessel be emptied, the sides will not fall together."[30]

Barrow draws a further argument in the same direction from the local motion of bodies within the universe. He stresses that these are always slow in comparison with the would-be instantaneous, automatic contiguity of surrounding matter that would inevitably follow the removal of any material body, if no space existed. Therefore, it would be impossible for a body to come to occupy the place left available by some other body. Hence, no motion would be possible: if no space exists, must "every flux be stopped, and every motion be put at rest or destroyed."[31]

Notice that Descartes and Barrow draw opposite conclusions from the same thought experiment, the only difference being their ideas of God. According to Descartes, although in principle God can void space of matter, still God's action would be immediately followed by the collapse of the vessel sides, and therefore God cannot in fact make space devoid of matter. According to Barrow, God can void space of matter; since the automatic conflation or collapse, if emptied of matter, of all the sides of any kind of closed rooms is absurd in logical as well as physical terms, therefore space must exist.

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[29] Barrow, *Lectiones mathematicae*, p. 157.

[30] Barrow, *Lectiones mathematicae*, p. 157.

[31] Barrow, *Lectiones mathematicae*, p. 157.

Interestingly, Barrow's answers to the arguments against the existence of space concludes with an explicit reference to the "physical experiments" that would support the existence of an empty (*vacuo*) space. They do not seem conclusive to Barrow. On a line similar to Hobbes's, Barrow argues that conclusions based on experimental demonstrations are not foolproof because other explanations may work as well. Arguments based on air-pump experiments, says Barrow, can be countered with specific, additional hypotheses about "subtle matter, circular motion, and infinite divisibility."[32]

Barrow's foregoing discussion (which I have shortened significantly) allows him to conclude that space exists, although he has not yet said anything about its nature. Barrow's own notion of space, closely linked to God's nature and attributes, does not stress God's omnipresence (as More did) but his omnipotence. Barrow's space is the reification of God's power to create or locate magnitudes anywhere according to His pleasure and free will. Barrow's space is nothing but "pure potentiality (*potentia*), mere capacity, capability to place (*ponibilitas*) magnitude, or … to place it in between (*interponibilitas*) [other magnitudes]".[33] Space, says Barrow, is not a figment of the human imagination (he is aiming at Hobbes's here, as we shall see), but a real thing, a "being" (*ens*) that falls under the general category of possibilities or capabilities, as real as are "sensibility" or "mobility".[34]

The reality of such a "being" derives from God's power of creating (or destroying) matter, anytime, anywhere. Before the creation of the world, when no

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[32] Barrow, *Lectiones mathematicae*, p. 158.

[33] Barrow, *Lectiones mathematicae*, p. 158.

[34] Barrow, *Lectiones mathematicae*, p. 160.

body existed anywhere, yet bodies could in principle exist according to God's will, as many of them as He liked, placed in any place He liked. Therefore, space —understood as the potentiality for locating or receiving bodies— was there ("*fiat spatium*"). Likewise, outside our (to Barrow, obviously finite) world, there are no bodies. Yet, God can obviously create bodies there. Therefore, there is an extracosmic space ("*datur spatium ultramundanum*"). Likewise, if divine power destroyed all matter within the walls of a room, then while nothing would be therein, yet it would be possible to place material things of appropriate dimension there. Therefore, there is space between the walls ("*datur spatium illis interjectum*").[35]

As something that exists as potentiality, the nature of Barrow's space is neutral or indefinite (*indefinita*) as regards its physical properties.[36] Moreover, Barrow's space is different from magnitude, "is not something ... endowed with actual measure (*dimensio*), actually extended". Therefore, the quantity of space cannot be determined immediately from itself, but only mediately by the measure of some real magnitude (a stretched line, for instance) occupying it.[37]

To close the chapter he has devoted to the nature of space, Barrow underscores the deep "affinity [*cognatio*] and analogy" between space and time. He characterizes it (without elaboration) by stressing that space is to magnitude as time is to motion as, or

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[35] Barrow, *Lectiones mathematicae*, p. 158-9.

[36] Barrow, *Lectiones mathematicae*, p. 159.

[37] Barrow, *Lectiones mathematicae*, p. 158, 159, 160; quotation on p. 159.

that “time is somehow the space of motion” (*ut tempus sit quodammodo spatium motâs*). [38]

BARROW AND HOBBES

Barrow stressed the theological dimension of his notion of space. His notion avoided the most important difficulties, signally including that it was not a new substance and had no magnitude, and therefore did not imply the infinity of matter. Furthermore, his notion of space most safely preserves divine prerogatives. Besides that it does not contradict in any way God’s ubiquity, it falls short of being something eternal, infinite, unproduced, and non-dependent on God, and therefore a challenge to God’s preeminence in the order of things. On the contrary, Barrow’s space proclaims and warrants His unbounded power to create bodies anywhere.[39]

Interestingly, Barrow’s stress on the theological virtues of his space is presented just before he turns to criticize Hobbes’s notion of space. According to Hobbes, the only really existing things are matter and motion, and therefore he cannot consider space to be a real self-subsisting entity. On the contrary, space is an imaginary notion, a *phantasma* produced in the human imagination by the real magnitudes of bodies outside us that impinge in our senses.[40] It is the image that remains in us when we remember body *simpliciter*, devoid of particular features. “Space is the image of body

[38] Barrow, *Lectiones mathematicae*, p. 165.

[39] Barrow, *Lectiones mathematicae*, p. 161.

[40] On Hobbes’s notion of space, see C. Leijenhorst, *The mechanisation of Aristotelianism. The Late Aristotelian Setting of Thomas Hobbes’ Natural Philosophy* (Leiden: Brill, 2002), 101-128. See also A. Malet, “The power of images: Mathematics and metaphysics in Hobbes's optics”, *Studies in History and Philosophy of Science*, **32** (2001): 303-333.

as body."[41] As C. Leijenhorst has recently pointed out, Hobbes seriously challenged an understanding of sense perception and imagination in which the human soul was able to freely work out its images. Instead, Hobbes considered imagination as mechanically determined by the material effects produced by external bodies. In this challenge, space is the representation of body in general and a central notion in our organization of the image of the world. As such, it becomes Hobbes's paradigmatic *phantasma*.[42]

Barrow stresses that hiss notion of space is "almost direct contrary to the definition of it which is delivered by Mr. Hobbes."[43] Barrow's critique of Hobbes's notion of space argues that space is not a phantasm but an object that feeds our imagination. It is something imaginable rather than an effect of our imagination. And Barrow points out that when space is some phantasma in our imagination, it cannot be the image of body as existing, but of a thing as possible: "When we conceive space we conceive that some magnitude may exist, ... Space is rather the idea of things as possible."[44] Actually, says Barrow, Hobbes characterizes space in terms of magnitudes that only exist potentially, not actually.

THE ROLE OF BARROW'S *MATHEMATICAL LECTURES*

Now, why was it important for Barrow to devote a long chapter of his Cambridge *Mathematical Lectures* to this rather rarified (if you allow me the pun) discussion? In my view the answer is to be found in the role Barrow intended for his *Mathematical*

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[41] Quoted in Leijenhorst, *The mechanisation of Aristotelianism*, p. 106.

[42] Leijenhorst, *The mechanisation of Aristotelianism*, p. 123, 107.

[43] Barrow, *Usefulness*, p. 179.

[44] Barrow, *Usefulness*, p. 179-180.

Lectures within the intelectual and political context of the early years of the English Restauration. As M. Hunter and others have shown, the extended and manifaceted reaction against "atheism" in Restoration England cannot easily be overestimated, although it is open to discussion what was then exactly meant by "atheism" and whether the dangers so acutely felt were real or largely imaginary.[45] Many contemporary thinkers voiced preoccupation with, and decided to fight against, what they saw as the increasing influence of Hobbesian and Cartesian mechanical natural philosophy—an influence that was deemed conducive to atheism and materialism. As is well known, the early-Restoration anti-Hobbesianism had in the Scargill affair one of its most dramatic moments, when the Cambridge don, Daniel Scargill, was expelled in 1668 from the University on the charge of teaching and endorsing Hobbes's ideas.[46]

For our purposes it is to be highlighted that a substantial body of literature was written during the Restoration in answer to two most dangerous philosophical theses. That matter is eternal and uncreated, was one of them. The second was that matter, by itself or helped by physical, *blind* active principles, produced the whole range of

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[45] M. Hunter, *Science and Society in Restoration England* (Cambridge: Cambridge Univ. Press, 1981), p. 162-87; 'Science and heterodoxy: An early modern problem reconsidered', in *Reappraisals of the Scientific Revolution*, D.C. Lindberg, R.S. Westman eds. (Cambridge: Cambridge Univ. Press, 1990), p. 437-60. See also M. Hunter, ' "Aikenhead the Atheist": The Context and Consequences of Articulate Irreligion in the Late Seventeenth Century' and N. Smith, 'The Charge of Atheism and the Language of Radical Speculation, 1640-1660', in *Atheism from the Reformation to the Enlightenment*, M. Hunter and D. Wootton eds. (Oxford: Clarendon Press, 1992), p. 221-54 and 131-58.

[46] A Cambridge don, Daniel Scargill was expelled in 1668 from the University on the charge of teaching and endorsing Hobbes's ideas. He submitted a six-page pitiful recantation hoping for his reinstatement as fellow of Corpus Christi College, which he did not get: *The Recantation of Daniel Scargill, Publickly made before the University of Cambridge* (Cambridge, 1669). On Scargill, see J.L. Axtell, 'The

physical effects without the direct and purposeful intervention of God, which was tantamount to the notion of a Godless material world.[47] Now, an autonomous nature, one that works without God's assistance, was a dangerous philosophical notion, as it was assumed to promote moral relativism, religious indifference, and political instability.[48] As Henry More put it, one of the causes of atheism is 'ignorance of the scantness and insufficiency of second causes'.[49] Hobbesians and other mechanical philosophers countenanced—or so it was perceived—a deterministic world, a clockwork wound up since creation in which God's ordinary providence was reduced to his continuously willing the world's existence. In contradistinction, many Restoration thinkers (including Boyle and Barrow) emphasized God's active role in keeping this world orderly working, which led them to de-emphasize the role of 'second causes' and to eliminate necessity from the 'natural' workings of nature. It is my suggestion that

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Mechanics of Opposition: Restoration Cambridge vs Daniel Scargill', *Bull. Inst. Hist. Res.* (1965) *38*, 102-11, and Mintz, *Hunting of Leviathan*, op. cit. (44), p. 50-2.

[47] Robert Kargon has documented the growing dissatisfaction with mechanical accounts of nature in the writings of Richard Baxter, Samuel Parker, and John Glanvill in his *Atomism* n. (3), p. 106-117.

[48] S.I. Mintz, *The Hunting of Leviathan. Seventeenth-Century Reactions to the Materialism and Moral Philosophy of Thomas Hobbes* (Cambridge: Cambridge Univ. Press, 1970), p. 69-88; R.L. Colie, *Light and Enlightenment. A Study of the Cambridge Platonists and the Dutch Arminians* (Cambridge: Cambridge Univ. Press, 1957); J. Redwood, *Reason, Ridicule and Religion. The Age of the Enlightenment in England 1660-1750* (London: Thames and Hudson, 1976); O. Mayr, *Authority, Liberty & Automatic Machinery in Early Modern Europe* (Baltimore, etc.: The Johns Hopkins Univ. Press, 1986); Kargon, *Atomism*, n. (3); J.R. Jacob, *Henry Stubbe, radical Protestantism and the early Enlightenment* (Cambridge: Cambridge Univ. Press, 1983); H. Guerlac, *Newton et Epicure* (Paris: Conferences du Palais de la Decouverte, 1963).

[49] *Antidote Against Atheism* (1712), p. 141, quoted by Mintz, *Hunting of Leviathan*, op. cit. (44), p. 88. See the bibliography on More in notes 40 and 45, and also C. Webster, 'Henry More and Descartes:

Barrow's *Mathematical Lectures* was a contribution to the enterprise of sanitazing natural philosophy generally, and mechanical philosophy very particularly, so that it could not be a weapon against a view of nature deemed essential to the preservation of religious and political order.

In fact, as mentioned before, Barrow's *Mathematical Lectures* contains a critical exploration of the notion of causality that introduces a supernaturalistic understanding of the production of all physical effects in nature. Barrow viewed the world entirely depending on God's will and power, a view of nature that also figures prominently in many of Barrow's religious sermons. Yet, even in his more sober *Mathematical Lectures*, Barrow argues that we cannot possibly know how God does organize the world, nor how he does take care of it--which implies that any attempt to understand the causal mechanisms behind the effects we perceive in nature is bound to fail. Therefore, Barrow concludes that true and sound natural philosophy cannot aspire to find out the secondary, actual, material causes of physical effects.

Barrow's stance in relation to causal physical explanations is part of his anti-metaphysical frame of mind. In introducing notions such as "magnitude" or "space" that could be the object of carefully worded definitions and characterizations in terms of specific properties or features, Barrow discusses them with explicit antimetaphysical disclaimers. He has no place for a formal definition of magnitude, but also denies the existence of any "primary notion and most essential property" of magnitude that would provide its "nature" or essence, and from whence all remaining properties would be deduced. In Barrow's judgment, there is no property to which a "primacy of order" does belong. Magnitude has many properties, but Barrow selects to explain those that

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Some new sources', *Brit. Jour. Hist Sci.* (1969), *4*, 359-77; R.A. Greene, 'Henry More and Robert Boyle on the Spirit of Nature', *Jour. Hist. Id.* (1962), *23*, 451-74.

are "most observable" (*sic*) and which are directly relevant to understanding mathematical hypothesis and principles.[50] Barrow repeats the same caveat when he comes to discuss space, under the umbrella of an strightforward disclaimer. He points out the obscurity of the subject, the many contradictory opinions it has prompted, avows his skepticism, and explicitly denies any claim to metaphysical certitude. Distrustful of metaphysical arguments, Barrow takes common sense and mathematical reasoning as the main guides to follow:

> I will take nothing for certain, will advance nothing for true in a case so difficult and slippery, nor assert anything confidently. But if … I am constrained to declare publicly what to me seems more perfecftly true, then I am not too averse to the [common] conceptions of men, nor willing to cast aside the sacred tenets of geometry.[51]

Barrow's alternative to natural philosophies à la Descartes o à la Hobbes, which explained the causal mechanisms of nature grounded on clear metaphysical principles, was his reform of the mixed mathematical sciences and his strong program of mathematization of natural philosophy. Barrow's *Mathematical Lectures* contain the innovative idea that mathematical principles or hypotheses need to be free from contradiction, but it is no longer needed that they be obviously true. The only requierement for acceptable hypotheses in the mathematical sciences is that it be "reasonable" or suggested by "plausible reasons".[52] "[T]rue Hypotheses", he says, are

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[50] Barrow, *Usefulness*, p. 134-5.

[51] Barrow, *Lectiones mathematicae*, p. 158.

[52] That hypotheses in natural philosophy need not be true a priori was of course a common feature in Cartesian natural philosophy, and Boyle's *About the Excellency and Grounds of the Mechanical Hypothesis* contains a forceful defense of the advancing of hypotheses which one cannot prove to be true

those "such as imply no inconsistence in themselves". The falsehood or inadequacy of hypotheses is defined consequently: "the falsehood of any hypothesis seems to be nothing but the conception or position of things as effected or existing, which cannot be effected or exist."[53] This novel understanding of mathematical principles broadens the field open to mathematical demonstration:

> Hence, also it follows, that demonstrations may be made of things, which never had existence any where; because it is sufficient for a demonstration to assume true hypotheses, i.e. such as imply no inconsistence in themselves.[54]

What is then the status of mathematical results correctly deduced from axioms that are not physically true? Barrow interpreted them as making up theories or mathematical constructs that, though of no use here in our world, describe 'imaginary' worlds which God in his omnipotence might have created. Conclusions may be physically true or false (it depends on the axioms being so), but they will always be "lawful", that is to say, mathematically unobjectionable. He supported his views with two illuminating examples, the Galilean law of falling bodies, and astronomical theories. The different astronomical systems or are in principle all equally valid because they describe as many imaginary worlds:

> yet because nothing hinders, but God may create such a world, where the stars will exactly agree with such motions; therefore the demonstrations depending upon such hypotheses are most true, and their astronomy true,

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a priori (*Works*, London, 1772, IV, p. 77). What is new in Barrow's discussion is that such hypotheses can be used as mathematical axioms.

[53] *Usefulness of Mathematical Learning*, n. (8), p. 109.

[54] Ibid., p. 110.

> not indeed of this world, but of some other which God can create. For God has given us the power of creating almost innumerable imaginary worlds in our thoughts, which himself, if he please, can cause to be real.[55]

The obvious corollary is that we need to test our theories against observations to make sure that they apply to *this* world. As I have shown elsewhere, this is a key notion in Barrow's geometrical optics. Barrow's "many worlds", perhaps utterly different from ours, we find explicitly bound up with his theological voluntarism. Because God's creation of our world was the result of a "wise free-choice", and not a necessary emanation, "He ... could have framed [the world] otherwise, according to an infinite variety of ways."[56]

CONCLUDING REMARKS

The *Mathematical Lectures* Barrow wrote and probably taught at Cambdrige in the mid 1660s shows his engagement in the most consequential philosophical debates of the times. There can be little doubt that he is an original contributor to the task of constructing new categories —magnitude, quantity, number, space, mathematical hypothesis, and so on— for the then fledgling experimental philosophy. It is telling the way he chooses and deals with his adversaries. In his discussion of the nature of space, Barrow makes passing references to notions entertained by Henry More and the 17th-

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[55] *Mathematical Works*, n. (8), p. 112. I have modified the translation provided in *Usefulness of Mathematical Learning*, n. (8), p. 111.

[56] 'Maker of Heaven and Earth', *Works*, n. (2), II, p. 183. The 'many worlds' debate has been often bound up with theological discussions on God's omnipotence. Cf. S.J. Dick, *Plurality of Worlds. The Origins of the Extraterrestrial Life Debate from Democritus to Kant* (Cambridge: Cambridge Univ. Press, 1984), p. 23ff; A. Funkenstein, *Theology and the Scientific Imagination*, op. cit. n. (12), p. 163ff.

century atomists, but spares them of detailed criticism. Descartes and Hobbes, on the other hand, are taken very seriously and carefully criticized. Much has been written about the debates between Hobbes and Boyle. I think I have shown here that Barrow engaged very seriously with Hobbes —as well as with Descartes— on the metaphysical front. In fact, Barrow's *Lectures* seem to me were meant to expose the flaws and weaknesses of their mechanical philosophies. Barrow's attack is two- or three-sided. On the one hand, he argues that it is useless to try to provide causal mechanical accounts of the works of nature. On the other, he criticizes basic notions of their systems, such as extension, space, and magnitude. More devastatingly, he casts doubts of principle, or method, on their attempts to secure unassailable metaphysical foundations for their systems. Furthermore, he provides an alternative to the systems of mechanical philosophy by way of his strong program of mathematization. Finally, Barrow's *Lectures* show that God is a major player in Barrow's arguments, clearly shaping his ideas of space and causality. His voluntaristic understanding of God allows him to make a distinction between physical truth and mathematical or formal truth, and thus facilitates Barrow's skepticism towards the mechanical philosophies of Descartes and Hobbes.