ENGLAND'S FORGOTTEN GALILEO A VIEW ON THOMAS HARRIOT'S BALLISTIC PARABOLAS

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Galileo's insight into the parabolic shape of projectile trajectories is commonly considered not only as having provided a major turning point in the history of ballistics, but also as having constituted a fundamental step towards the establishment of classical mechanics. In fact, from a modern perspective, the parabolic trajectory is closely associated with three of the most fundamental principles of classical mechanics, the law of inertia, the law of fall, and the superposition of motions without interference.

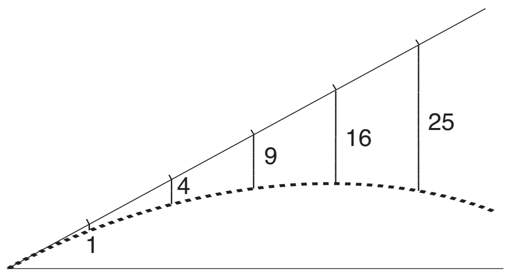

Fig. 1: Construction of a projectile trajectory in classical mechanics.

According to classical mechanics, the projectile trajectory results from a composition of two motions, an inertial motion along the line of the shot, and the accelerated motion of free fall vertically downwards. Given the knowledge of the laws governing these two fundamental kinds of motion, points on the trajectory can be geometrically constructed by considering the distances traversed by the two motions in equal intervals of time. Figure 1 illustrates such a construction. To represent the uniform inertial motion along the line of the shot, equidistant points are plotted on the oblique line, marking the distances traversed in equal intervals of time. From these points, the distances the mobile traverses in free fall in the time that has passed since the beginning of the shot are measured vertically downwards. Since the space traversed in free fall grows quadratically in time, these distances increase according to the sequence of square numbers: 1, 4, 9, etc. The resulting points mark the actual positions of the projectile after equal intervals of time. The trajectory, represented as a dotted line in Figure 1, is drawn by joining these construction points smoothly.

This construction appears to be so immediately plausible that one is tempted to assume that whoever conceives of the projectile trajectory as a parabola resulting from a composition of two motions, must understand it in this way and hence also attain the insight into the law of inertia and the law of fall underlying the construction. Accordingly, the fact that Galileo, in his late work on mechanics, the *Discorsi*, formulated a classical result, namely the parabolic trajectory, is usually understood to imply that he was already working within the framework of classical mechanics. Even evident deviations in his works from the reasoning expected according to classical physics are usually not understood as indicating that Galileo's arguments were actually *not* rooted in the framework of that science.

An example of such a deviation is provided by a striking gap in the deductive structure of the *Discorsi*, the major work on mechanics of the mature Galileo, after all. Galileo derives the form of the projectile trajectory only in the case of horizontal projection, by composing the horizontal component with the vertical motion of fall –precisely according to the construction presented above. For the case of oblique projection, however, he just *states* that the resulting trajectory would likewise be a parabola, without offering any proof for this statement.[1] In the light of the fact that, in classical mechanics, the trajectory of oblique projection follows from the same construction as that for horizontal projection, this omission is hard to understand. In classical mechanics, the case of horizontal projection is, after all, merely a special case of the more general class of projections in any direction. However, if one takes into account not only Galileo's published works but also the numerous unpublished manuscripts documenting his ongoing research, one can indeed, as I shall illustrate in the following, identify clues illuminating why oblique projection represented a problem for him.

[1] Galileo (1968, Vol. VIII, p. 296) claims, but does not prove, that a bullet shot at a given angle would traverse the reverse path of a bullet shot horizontally and hitting the ground at the same angle. The deficiency in Galileo's argument was already pointed out by Descartes (1964 ff., Vol. II, p. 387, letter no. 146) in his famous critique of the *Discorsi*, and later discussed by Wohlwill, E. 1884, pp. 111 f.

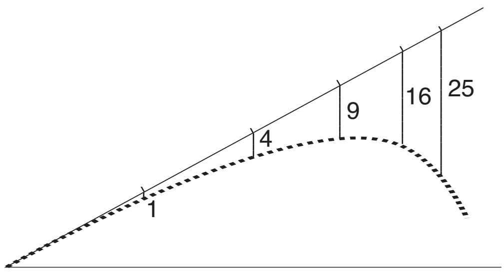

Fig. 2: Construction of a projectile trajectory based on an interpretation of f. 175v, Galileo MS 72.

Figure 2 illustrates the construction of a projectile trajectory for oblique motion as can be reconstructed from a drawing found in Galileo's manuscripts.[2] In this representation the bullet is projected in the lower left corner. After being projected it participates in two motions. One is the motion along the line of the shot, the other is the motion vertically downwards. The construction thus looks very similar to that of classical mechanics (see Figure 1). There is, however, one major difference between Galileo's construction and that of classical mechanics: the motion along the line of the shot in Galileo's construction is not the uniform inertial motion of classical mechanics. Rather, this motion is decelerated. The spaces traversed along the oblique line in equal intervals of time decrease in such a way that the resulting motion behaves like a reversed motion of fall. The motion along the line of the shot therefore appears to be modeled in analogy to the motion along an inclined plane. From the perspective of classical mechanics this conception is fallacious. Only in the case of horizontal projection does the result coincide with that of classical mechanics, since in that case the inclination of the plane is zero and no deceleration due to gravity occurs.[3]

Why did Galileo adhere to such a strange idea? Was he just infatuated with inclined planes as others claimed he was with circles? And if so, was it

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[2] Folio 175v of MS 72, Biblioteca Nazionale, Florence. Naylor (1980, pp. 557–561) was the first to interpret this drawing as a theoretical analysis of oblique projection. Damerow, Freudenthal, McLaughlin, and Renn (1992, pp. 206–209) follow Naylor but offer a different reconstruction of the conceptions underlying the construction. Here, I follow the interpretation given by the latter.

[3] Another example for such a deviation of the reasoning on projectile motion in Galileo's work from that expected according to classical mechanics points in the same direction. In the dialogue part of the *Discorsi* it is argued that, due to the curvature of the earth's surface, the line of the shot would actually have to be considered as inclining even in the case of horizontal projection, so that the motion along this line cannot be uniform. Only the smallness of this effect is mentioned in order to refute the objection (Galileo, G., 1968, Vol. VIII, pp. 274 f.). See also Wohlwill, E. 1884, pp. 112 f.

then just a mere coincidence that Galileo treated the case of horizontal projection correctly and thus hit upon what was to become a key insight of classical mechanics?

If one conceives of scientific ideas as being merely a product of individual thinking –great ideas as the product of genius, and lousy ideas as the product of infatuations that may affect even a genius– virtually no other explanation remains. However, if one takes into account the shared knowledge on the basis of individual thinking, what at first sight appears to be merely the individual blunder of a hero of science, may actually become plausible as an expression of a differently structured body of knowledge. From this perspective, the emergence of classical mechanics would hence not have to be explained as being due to an accidental discovery, but could be accounted for as the result of a transformation of the shared knowledge underlying also Galileo's thinking.

Are there any indications that Galileo's apparently eccentric idea of constructing the trajectory of oblique projection by means of an inclined plane is actually not just an individual idiosyncrasy but strongly suggested by the shared knowledge of his time, a knowledge possibly structured by other principles than those of classical physics? Evidently, this question cannot be answered by looking at Galileo's work alone, and has, accordingly, been neglected by Galileo scholars. What did early modern practitioners and theoreticians of artillery –Leonardo da Vinci, Giusto Aquilone, Paulus Puchner, Sebastian Münster, Daniel Santbech, William Bourne, Niccolò Tartaglia, Alessandro Capobianco, Luys Collado, or Diego Uffano, to name a few– think about projectile motion? Did they each have their distinct individual views, or is it possible to recognize structural similarities in their conceptions, revealing a body of non-classical shared knowledge? A systematic study of such similarities has hardly begun. Here, I would like to offer a glimpse at the work on projectile motion of one such contemporary of Galileo, the English natural philosopher Thomas Harriot, using a reconstruction of his work from the extant manuscripts.[4]

Harriot lived from 1560 to 1621. He filled more than 8000 manuscript pages with notes on various topics of contemporary mathematics, natural philosophy, and engineering, but did not publish any of his scientific achievements. Though Harriot eventually became familiar with Galileo's astronomical work through the latter's publications, his work on ballistics has to be regarded as independent of Galileo's, no personal contact between the two being known, and Harriot's work having been completed long before Galileo published on ballistics.

[4] A comprehensive reconstruction of Harriot's work on ballistics is presently carried out by the author. This contribution refers to a few preliminary results.

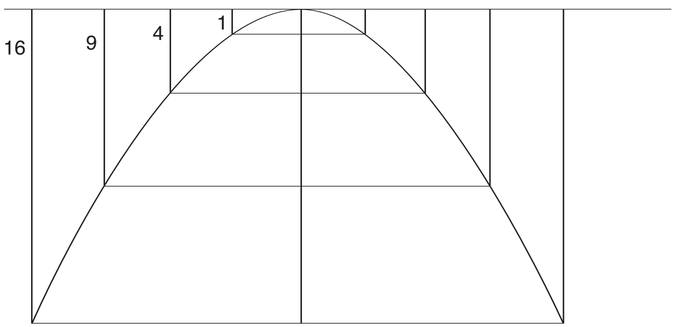

Fig 3: Folio 67r, BL Add MS 6789.

One of the folios in Harriot's manuscripts bearing notes on ballistics is preserved as f. 67r, Add MS 6789, in the British Library. On its upper part, there is a drawing of the curve reproduced in Figure 3. Below this drawing Harriot noted:[5]

> The species of the line that is made upon the shot of poynt blanke is as is here described & is a parabola as of the upper randons.

The "shot of point blank" thereby denotes the horizontal shot, while with the "upper randons" Harriot refers to the shots at an elevation above the horizontal. The trajectory is evidently constructed in the manner previously described, i.e. by composing motions traversed in equal intervals of time. Along the horizontal, equal distances are marked, thus representing a uniform motion. The lengths of the verticals obviously represent the motion of fall and grow quadratically as Harriot noted by writing down the numbers 1, 4, 9, and 16.

In short, Harriot's construction and the accompanying text, which must have been composed before 1621, the year of his death,[6] document his knowledge of the law of fall and of the parabolic shape of the projectile trajectory to the same extent as is known from Galileo's *Discorsi* of 1638. In the light of this document alone, it would thus seem to be justified to consider Harriot the Galileo of England. Indeed, if only this single document were known, Harriot could be credited as much as Galileo with the foundation of the classical theory of ballistics.

On closer inspection, however, it turns out that Harriot's construction not only produces the same insights but also displays the same weaknesses as Galileo's exposition in the *Discorsi*. In fact, it is only in the case of horizontal projection that the depicted parabola results from Harriot's construction. On this folio, at least, the parabolic shape of the trajectory for the case of oblique projection is only claimed but not proven.

[5] British Library Add MS 6789, f. 67r.

[6] On the basis of Harriot's handwriting, Shirley (1983, p. 261) dates these notes to 1607.

While Harriot did not publish on ballistics, he did leave us with many more manuscripts than Galileo, allowing us to reconstruct how he thought about oblique projection. In particular, the folio previously mentioned turns out not to be a disparate fragment, but rather part of a larger group of folios dealing with projection at arbitrary angles. Let us take a look at another one.

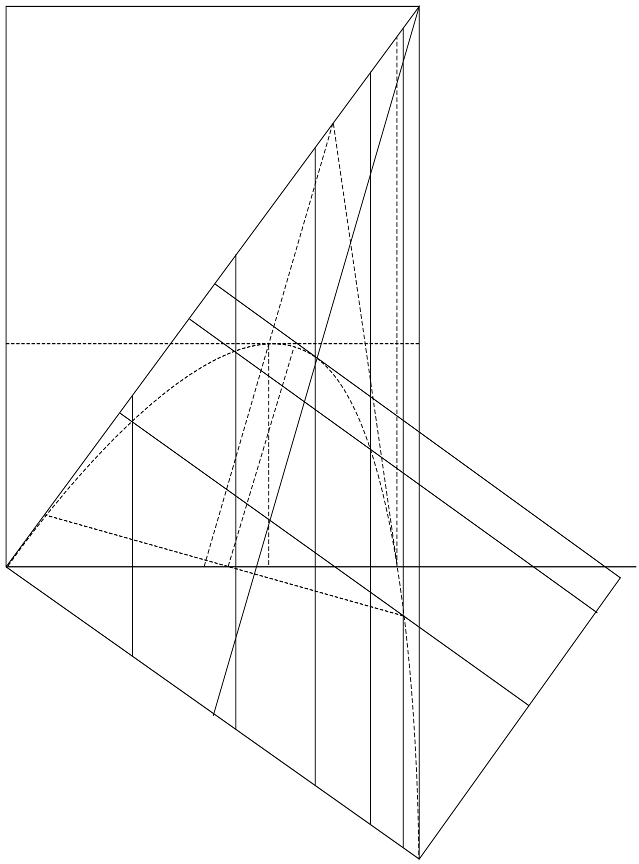

Fig. 4: Folio 64r, BL Add MS 6789.

The drawing on f. 64r, Add MS 6789, reproduced in Figure 4, illustrates a shot at an elevation above the horizontal (here at an angle of about 53°). The dotted curved line represents the trajectory, the oblique line tangent to it at its origin represents the line of the shot. From points on this line in decreasing distances, lines are drawn vertically downwards. The distances marked on these vertical lines from the line of the shot to the trajectory are of increasing length. Both, the deceleration of the motion along the line of the shot, as well as the acceleration of the motion along the vertical, obey a quadratic law. This suggests that also in Harriot's case, the motion along the line of the shot was conceived in analogy to the motion along an inclined plane, just as it has been the case in Galileo's construction.[7]

It thus seems that the use of the inclined plane in order to construct the trajectory of oblique projection was not an eccentric idea of Galileo, but rather a plausible option for anybody who, at that time, attempted to obtain the projectile trajectory from the composition of the motion along the line of the shot and the vertical motion of fall.

There is even stronger evidence to support the interpretation that Harriot's construction makes use of the inclined plane. As one can easily see in Figure 4, there are further construction lines drawn perpendicularly to the line of the shot, giving rise to triangular structures setting the distances traversed in the motion along the oblique in relation to those traversed along the vertical. Without going into details of the construction, I would like to mention that these structures assure that the motion along the oblique does indeed obey the law of the inclined plane.[8]

Harriot did not only construct these curves, he also analyzed their mathematical character, finding that they are indeed parabolas.[9] However, they

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[7] Lohne (1979, pp. 236 f.) interprets the deceleration along the oblique occurring in Harriot's trajectories as being due to air resistance. Although Harriot did indeed consider motion through a medium to be decelerated according to a quadratic law, this interpretation is untenable. As I will explain below, the deceleration of the oblique motion depends on the angle of elevation in a specific way, supporting the interpretation in terms of the inclined plane. In accordance with his understanding, Lohne interprets the drawing on f. 67r previously discussed, as representing the trajectory for the case that air resistance is neglected (Lohne, J. A., 1964, p. 19), obviously ignoring the fact that the folios are related and the drawings on them illustrate the trajectory for different angles of elevation.

[8] The law of the inclined plane states that the acceleration of a motion along the plane equals $\sin\alpha$ times the acceleration of free fall, where α denotes the angle of inclination. That this law holds in Harriot's construction can be seen as follows. The distances marked on the line of the shot by the vertical lines decrease according to the sequence of odd numbers, i.e. they are 11, 9, 7, 5, 3, and 1 units wide, so that a square law results when adding them up from above (1, 4, 9, etc.). In addition to this first motion along the line of the shot, Harriot considers a second one represented by the distances marked on the line of the shot by the lines perpendicular to the latter. In comparison to the first motion, this second motion is doubly decelerated, i.e. the spaces traversed in succeeding equal intervals of time are 10, 6, and 2 units, the double of the last three distances of the first motion. Then the doubly decelerated motion proceeds downwards again, traversing the same distances in reverse order. The difference of the spaces traversed by these two motions grows according to a square law as 1, 4, 9, etc., i.e. exactly as the motion along an inclined plane. Now consider the right triangles having the line segment representing the differences of the spaces traversed by the two motions as one leg, a line segment perpendicular to the line of the shot as the other, and a vertical line segment as the hypotenuse. The lower corners of these triangles are taken to be points on the trajectory, i.e. the hypotenuses represent the spaces fallen in free fall. But as they are as $1/\sin\alpha$ to the opposite legs representing the distances traversed along the inclined plane, where α denotes the elevation angle, the law of the inclined plane is satisfied.

[9] Besides many folios documenting Harriot's attempts at such a proof, f. 69r, BL Add MS 6789 bears the ultimate proof. For a transcription of this folio, see Lohne, J. A., 1979, pp. 258 f.

are tilted at an angle depending on the elevation of the shot.[10] Furthermore, Harriot used the composition of motions involving the inclined plane to solve problems that Galileo was also concerned with. In particular, like Galileo, Harriot was interested in solving the "gunner's question" of how the range of a shot depends on the gun's angle of elevation. Now, consistently applying the construction just explained to shots at different angles, a plausible answer to the gunner's problem may indeed be found.

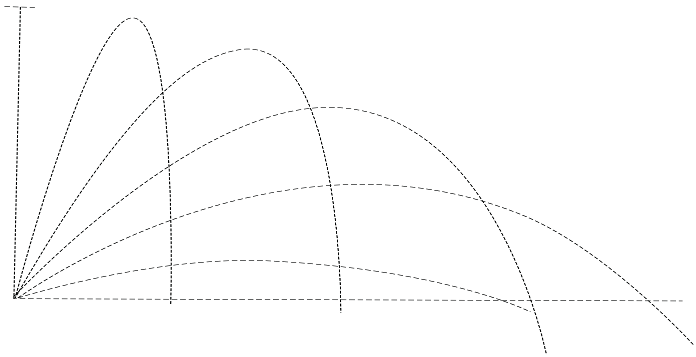

Fig. 5: Folio 216v, BL Add MS 6788.

Consider the drawing reproduced in Figure 5. There are five trajectories drawn at the angles of 15, 30, 45, 60 and 75 degrees. They differ in their appearance, particularly as regards the range of a shot. At first sight it is not clear how these trajectories were obtained since no construction lines are visible. On illuminating the folio with raking light, however, an abundance of construction lines carved into the paper but not drawn in ink become visible, some of which are redrawn in Figure 6. On closer analysis these lines reveal that the basic construction principles for all five trajectories are exactly those previously outlined.[11]

[10] The tilting angle β is given by $\tan\beta = \sin\alpha\sin\beta/(1+\sin^2\alpha)$, where α denotes the elevation angle; see Lohne, J. A., 1979, p. 238.

[11] Although the resulting curves obey the same principles as those on f. 64r, BL Add MS 6789, their actual construction is different. In Figure 6 there are four obliques representing the line of the shot for the respective elevations and one vertical. The concentric circles divide these lines into equal sections. The intersection points would thus represent a uniform motion. On the vertical line, the distances a body falls in a given time are measured from these intersection points downwards and marked with horizontal bars. From the first intersection point beginning from below, the distance a body falls in one interval of time is measured, from the second point that fallen in two intervals of time, and so on. Accordingly, these distances increase quadratically. The same distances are also measured vertically downwards from the respective intersection points of the circles with the obliques. From the endpoints of the vertical line segments thus obtained, a line is drawn perpendicular to the respective line of the shot. The intersection point of this perpendicular with the line of the shot marks the position the motion along the line of the shot reaches in the given time. From this point vertically downwards the respective distance of fall is laid down. In this way the points on the trajectory are generated.

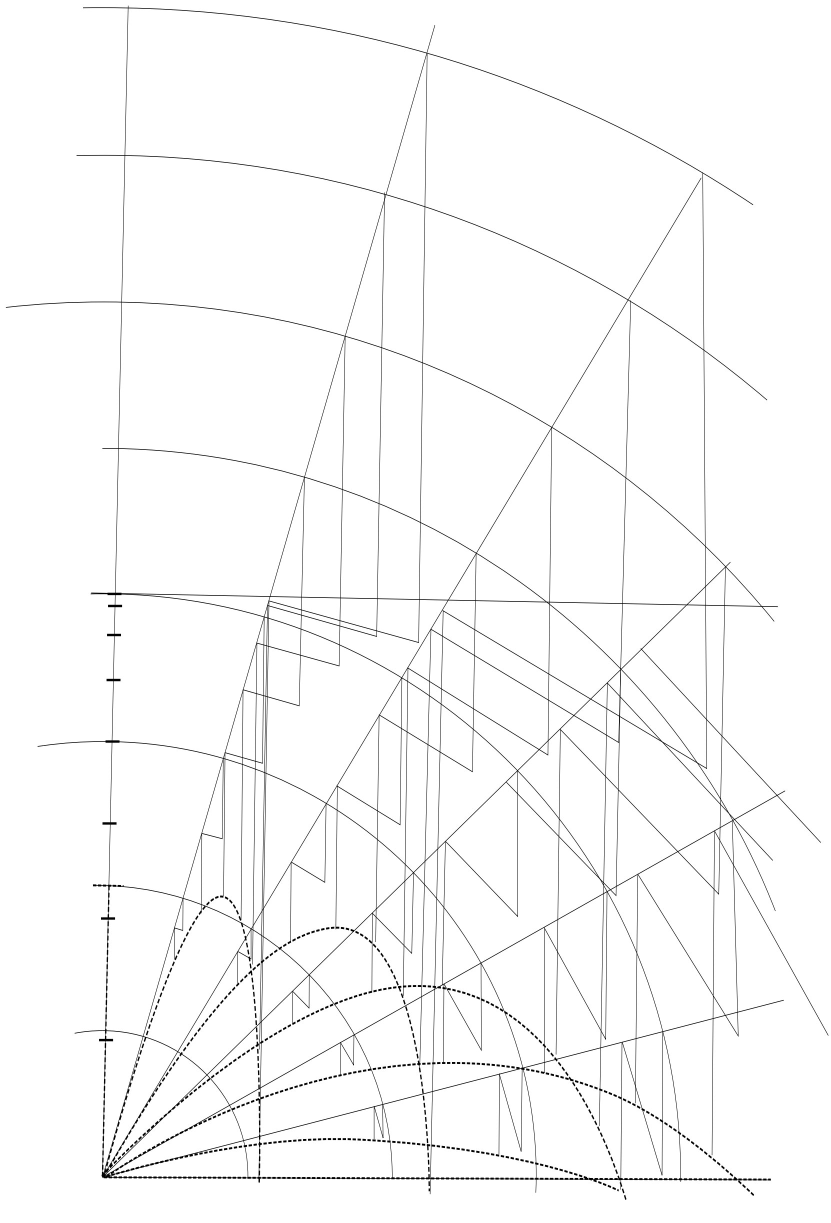

Fig. 6: Folio 216v, BL Add MS 6788
(construction lines not drawn in ink are represented as thin lines). While in the construction on f. 64r, BL Add MS 6789, the vertical distances were adapted to the oblique ones in order to satisfy the law of the inclined plane, here the oblique distances are adapted to the fixed vertical distances of fall, thus allowing for a comparison of the trajectories for shots at different angles.

Obviously, the construction also implies the existence of an elevation angle of maximum range. Harriot was even able to calculate this angle, determining the maximum range to be at an elevation of about 27°55'.[12]

Not only has Harriot pursued this line of thought further than Galileo, enabling its various consequences to be studied, Harriot's manuscripts also provide an insight into the origins of this conception of projectile motion.

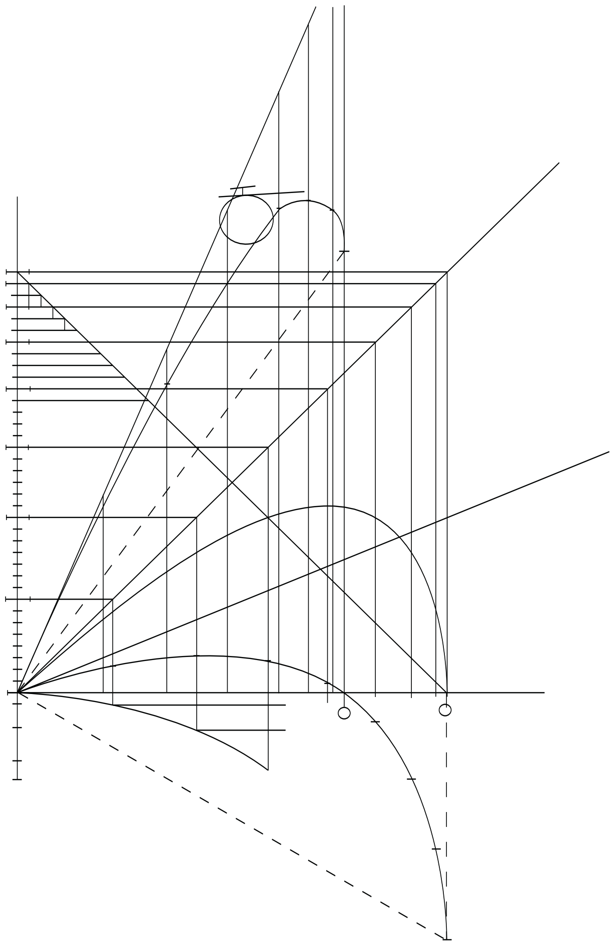

Fig. 7: Folio 4r, BL Add MS 6789.

Figure 7 shows a drawing from what probably represents an earlier period of Harriot's occupation with projectile motion. While the basic idea of the composition of two motions is the same as in the later constructions, nei-

[12] British Library Add MS 6788, f. 165v.

ther the correct law of fall, nor the law of the inclined plane are actually applied in the construction. The fact that even the motion along the horizontal is decelerated suggests that it is not the specific idea of an analogy to the inclined plane that constitutes the basis for this construction. Rather, it appears to be the natural exhaustion of the violent motion that leads to a deceleration along the line of the shot. When this violent motion has ceased, only the natural motion remains and the projectile falls vertically downwards, as one can see in Figure 7 in the case of the steepest shot.

This example may serve to illustrate what a more thorough analysis of numerous extant manuscripts amply confirms: that Harriot's constructions of trajectories, and in particular also his construction with the help of an inclined plane, can be understood as specific implementations of the Aristotelian dynamics of violent and natural motion. In fact, from this perspective, the inclined plane allows to specify in a physically plausible and mathematically tractable way the decrease of the violent motion and the way it depends on the angle of elevation.

As is well known, Galileo –even in his *Discorsi*– continues to make use of the concepts of violent and natural motion. While this is normally treated as nothing but a traditional way of speaking, on the background of the analysis just given it becomes evident that this has deeper implications: in fact, the deviations of Galileo's arguments from classical mechanics become understandable as the expression of a non-classical conceptual organization of knowledge that surfaces in the use of this traditional terminology. The knowledge, which Galileo shared with his contemporaries, is still rooted in the dynamical conceptions of Aristotle, but also comprises the experiences accumulated by the practitioners of ballistics, for instance the insight that there is an angle at which the shots obtain a maximum range.

The analysis of this shared knowledge cannot be covered by this limited contribution, whose aim is to point to the existence of this knowledge, which becomes strikingly visible in the remarkable similarities between the otherwise unrelated work of Harriot and Galileo. These similarities can hardly be explained by the traditional paradigm of influence and reception but have to be understood as the outcome of common challenges and shared means to address them. In short, studying the science of Thomas Harriot also entails learning about the roots of Galileo's contributions to classical mechanics.

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